THE EXTENT OF THE SIMILARITY BETWEEN BELOW RESONANCE AND ABOVE RESONANCE OPERATION OF STANDARD CIRCULATOR JUNCTIONS

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ABSTRACT

Starting from standard expressions for the tensor permiability components κ and μ, it can readily be shown that every circulator junction with $p = 4\pi M_S\gamma/\omega < 1$ has two low frequency modes of operation ($kR \simeq 1.84$) corresponding to below resonance and above resonance operation. In this contribution the extent of the similarity between those modes of operation is explored.

I Similarities - the frequency dependence of the susceptance

The existence of two low frequency modes of operation of standard circulator junctions is made most intuitive by considering the behavior of the tensor permiability components κ and μ in the very low field and very high field limits. In particular it is readily shown from expressions for these tensor permiability components that in the limit that the internal magnetic field $H_i \to \infty$ one has a reciprocal device with an effective permiability $\mu_{eff} = 1$. That is one has the same reciprocal device one has for H=0 except that μ_{eff} is somewhat less than one in this case. Consequently, for a given junction geometry and ferrite or garnet material, a nonreciprocal circulator with, to first order, the same center frequency and bandwidth can be obtained in one of two ways: by increasing the magnetic field from zero (below resonance operation) or by decreasing the magnetic field from infinity (above resonance operation). The direction of circulation will be opposite for the two cases.

The usual expressions for the tensor permiability components μ and κ of a saturated material are

$$\mu = 1 + \frac{4\pi M_S H_i \gamma^2}{\gamma^2 H_i^2 - \omega^2} \qquad (1)$$

$$\kappa = \frac{4\pi \gamma M_S \omega}{\gamma^2 H_i^2 - \omega^2} \qquad (2)$$

where M_S is the saturation magnetization of the ferrite or garnet material, H_i is the internal magnetic field, and $\gamma = 2.8$. Furthermore, the effective permiability μ_{eff} is given by

$$\mu_{eff} = \mu(1-\kappa^2/\mu^2) \quad . \qquad (3)$$

Clearly in the limit $H_i \to \infty$, $\kappa=0$ and $\mu=1$. From (3) it follows that $\mu_{eff} = 1$. In the very large field limit any junction containing a ferrite or garnet material will behave exactly as if the junction contained a reciprocal dielectric material with the same dielectric constant.

Consider now the low field limit. In this limit the material won't be saturated so that equations (1) and (2) must be replaced by

$$\mu = \mu_o + (1-\mu_o)\left(\frac{M}{M_S}\right)^{3/2} \qquad (4)$$

$$\kappa = -\frac{4\pi \gamma M}{\omega} \qquad (5)$$

where the demagnetized permiability μ_o is given by

$$\mu_o = \frac{2}{3}\left[1 - \left(\frac{\gamma 4\pi M_S}{\omega}\right)^2\right]^{1/2} + 1/3 \ . \qquad (6)$$

The magnetization M is less than the saturation magnetization M_S and increases from zero with applied field. Equation (3) is still applicable so that for H=0 (M=0), $\mu_{eff}=\mu_o<1$. Consequently, the low field and high field states differ only by having somewhat different effective permiabilities. This is illustrated experimentally in Fig.1 where return loss vs. frequency is plotted for a stripline circulator junction using 1/2" diameter X506 disks ($4\pi M_S$=500g., $p\simeq.3$) in zero field and a field of 5 kilogauss.

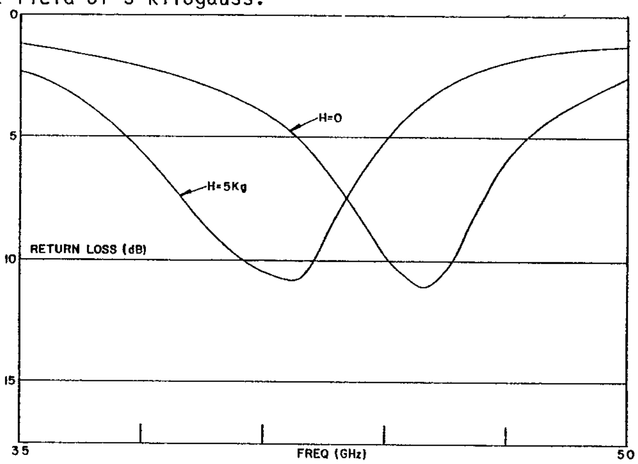

Fig.1 Return loss vs. frequency for a circulator junction in zero field and a field of 5Kg.

The reciprocal behavior in both cases is indicated by the minimum return loss of about 10dB (VSWR≃2/1). The minimum VSWR of such a reciprocal junction is 2/1. The high field curve lies below the zero field curve becuase $\mu_{eff}<1$ (H=0).

Clearly a circulator with about the same center frequency - determined approximately by the relation $kR = 1.84$ and bandwidth can be obtained in one of two ways: by increasing the magnetic field from zero or decreasing the magnetic field from infinity. This is demonstrated experimentally in Fig. 2 where return loss vs. frequency is plotted for a circulator junction using 1/2" diameter X-506 disks ($4\pi M_S$ = 500g., $p\simeq.3$) biased above and below resonance. The center frequency is somewhat lower for the above resonance case because μ_{eff} is larger above resonance. In order to be able to say more we must consider equations (1) and (2) in the limit $\gamma H_i >> \omega$. In this limit they become

$$\mu \simeq 1 + \frac{4\pi M_S}{H_i} \qquad (7)$$

$$\kappa \simeq \frac{4\pi M_S \omega}{\gamma H_i^2} \qquad (8)$$

$$\mu_{eff} \simeq \mu = 1 + \frac{4\pi M_S}{H_i} \qquad (9)$$

It is clear from (5) and (8) that κ has the opposite sign for the below and above resonance cases so that the direction of circulation will be opposite for these two

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modes of operation. Furthermore from (6)and (9) $\mu_{eff}<1$ (below resonance) and $\mu_{eff}>1$ (above resonance) so that the operating frequency will be lower in the latter case.

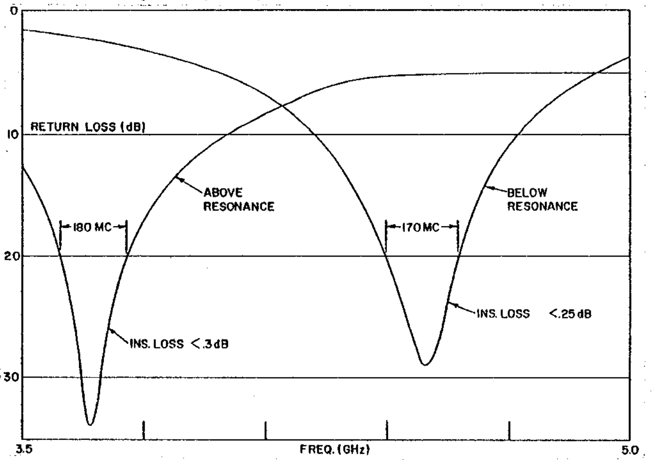

Fig.2 Return loss vs.frequency for a circulator junction biased below and above resonance.

The bandwidth is proportional to $\sqrt{\mu_{eff}}$ and consequently will be greater in the above resonance case. The absolute bandwidth in GHz is very nearly the same for the two cases. These conclusions are confirmed experimentally in Fig.3 where return loss vs.frequency is plotted for a circulator junction using 1/2" diameter G1600 disks($4\pi M_s$=1600g.) biased above resonance.

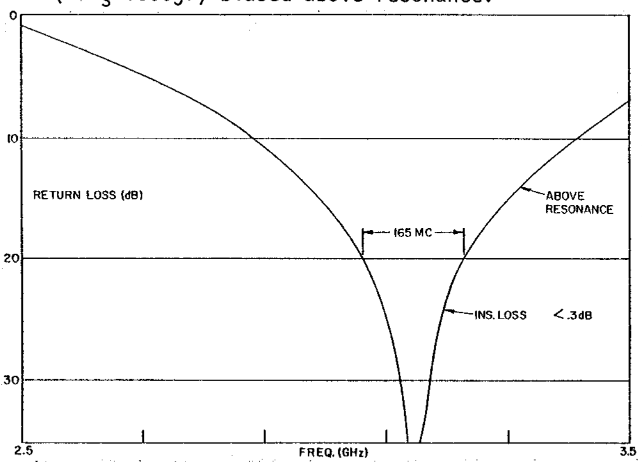

Fig.3 Return loss vs.frequency for a circulator junction biased above resonance.

The use of larger magnetization lowers the above resonance center frequency while the absolute bandwidth in MHz remains essentially constant. Choosing a larger value of magnetization increases μ_{eff}, lowers the center frequency and increases the relative bandwidth.

So far the similarities between below resonance and above resonance operation have been emphasized. These can be summarized by saying that the susceptive part of the equivalent admittance in the two states is very nearly the same once the difference in the effective permiability μ_{eff} has been taken into account. What about the conductive part? It is here that significant differences between the below resonance and above resonance states emerge. The behavior of the equivalent conductance is intimately related to the behavior of κ. From (5) κ is inversely proportional to frequency for the below resonance mode of operation while from (8) κ is proportional to frequency for the above resonance mode of operation. Consequently, we would expect to find the frequency dependence of the conductance G to be a principle difference between the below and above resonance modes of operation.

II Dissimilarities - the frequency dependence of the conductance

The frequency dependence of the conductance can be calculated using series expansions for the eigenadmittances of 3 port stripline circulators[1,2] in combination with expressions for the equivalent admittance(complex gyrator admittance)of these devices.[3,4] It will be recalled that this is a complex quantity with the property that if a 2 port network can be found which matches into this admittance, then the same 2 port network connected at each port of the circulator will match the circulator. Since the notion can be generalized to reciprocal networks, the term equivalent admittance will be prefered here. Bosma has given an expression for the conductive part G based on the lowest order terms in the eigenadmittance expansion.[2] He finds that

$$G = \frac{2}{\sqrt{3}\zeta}\,\frac{\kappa/\mu}{kr} \tag{10}$$

where ζ depends on physical dimensions and material constants, r is the radius of the garnet or ferrite disk, and $k=2\pi/\lambda$ with λ being the wavelength. Clearly from 4),5),7),8) and 10) G should be inversely proportional to ω^2 below resonance and independent of frequency above resonance. This result is in clear conflict with the experimental fact that G is nearly independent of frequency below resonance.[4,5] This fact in turn strongly suggests that G will be nearly proportional to ω^2 above resonance and that will be the working hypothesis of this paper.

It can be shown that the improper frequency dependence of G resulting from 10) is due to the failure to consider the next higher order terms in the Bosma-Davies, Cohen expansion for the eigenadmittances. In Fig.4a the theoretical conductance vs.frequency is given for the circulator junction of Figs.1 and 2 below resonance including only lowest order terms (dashed line) and including next higher order terms (solid line).

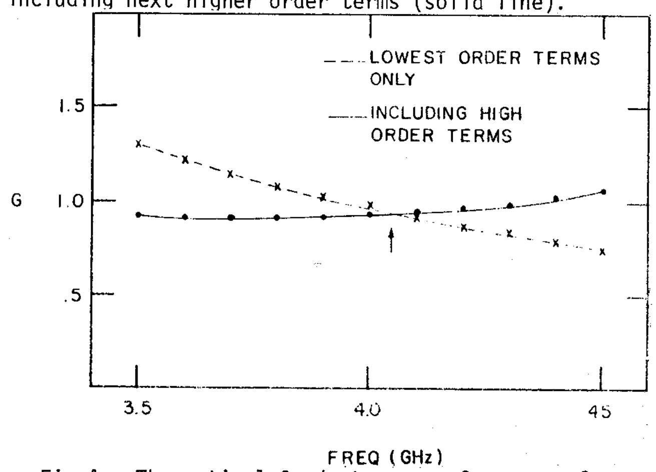

Fig.4a Theoretical Conductance vs.frequency for a circulator junction biased below resonance.

Including still further terms has a negligible effect. Including higher order terms removes most of the frequency dependence bringing us into agreement with the experimental situation. In Fig.4b similar theoretical curves are given only in this case above resonance. Now including higher order terms changes a nearly frequency independent conductance into one with a frequency dependence somewhat stronger than ω^2. The arrows in Figs.4a and 4b indicate the frequencies at which the susceptance is zero. These center frequencies lie somewhat lower then the experimental center frequencies of Fig.2.

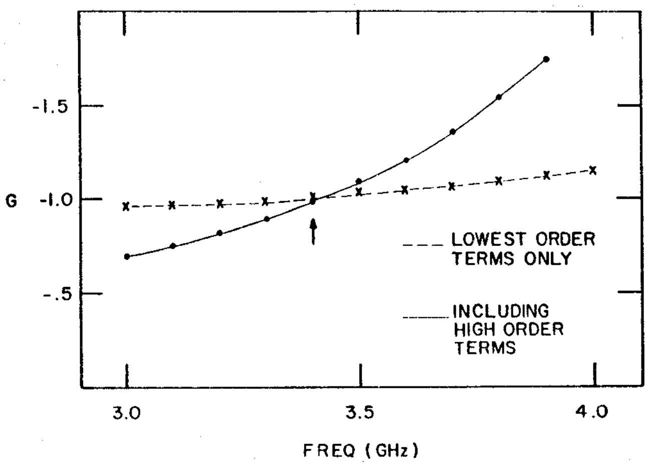

Fig.4b Theoretical conductance vs. frequency for a circulator junction biased above resonance.

III A Circuit model for the above resonance circulator

It has been proposed that the above resonance circulator has a conductance nearly proportional to ω^2. However at microwave frequencies one prefers to work in terms of the variable $t=\tan\Theta$. In terms of this variable the conductance is nearly proportional to t provided that the electrical length $\Theta=\pi/4$ at the center frequency of the junction. The equivalent admittance Y_{eq} of the above resonance circulator is given approximately by

$$Y_{eq} = G_0\tan\Theta - j\,Y_0\cot 2\Theta$$
$$= G_0 t + j\frac{Y_0}{2}\left\{t-1/t\right\} \qquad (11)$$

where G_0 depends on magnetic field strength. This is a quite surprising result since Y_{eq} has the same form as that for the simplest unbalanced line 2 branch lumped element directional coupler with capacitors connecting the various ports, provided that the variable t is replaced by the variable ω(see Fig.5).[6]

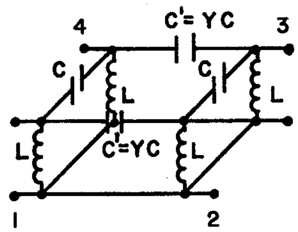

Fig.5 Simplest 2 branch unbalanced line lumped element coupler.

Networks which are useful for broadbanding one should be useful for broadbanding the other. In both cases Y_{eq} is not a P.R.function which is not too surprising since it is a mathematical construct which reduces a multi-port matching problem to a simpler one port matching problem.[6] If $L = 1 + \sqrt{2}$, $C = 1$, $Y = \sqrt{2}$ then the circuit of Fig.5 is a matched hybrid at ω=1 with

$$Y_{eq} = \omega + j(1+\sqrt{2})\left\{\omega - 1/\omega\right\} . \qquad (12)$$

IV Conclusions

To summarize, the susceptive part of the equivalent admittance is very similar for the below and above resonance modes of operation - particularly if account is taken of differences in the effective permiability μ_{eff}. On the other hand, whereas the conductance is nearly frequency independent below resonance, it varies approximately as $\omega^2 \approx t$ above resonance. This will tend to make broadband matching more difficult. To obtain this sort of frequency dependence, second order terms in the Bosma-Davies, Cohen expansion for the eigenadmittances must be included. The equivalent admittance of the above resonance circulator has the same form as that of the simplest unbalanced line 2 branch lumped element directional coupler with the four ports connected by capacitors, provided that the Richards' variable t is replaced by the frequency variable ω.

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